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Bicycle's Pendulum

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Abstract: **Bicycle wheel isn't necessary a mean transport, when it move freely it can be regarded as a pendulum which combines among properties of bicycles and pendulums at the same time, it's general construction and methods of building is similar to bicycles but it behaves as pendulums, it can make rotational, oscillation, transitional and random motion , it's time of full oscillation can be calculated easily and it is in direct relation with the length of rods, angle of oscillation and number of rods, using this time and pendulum's properties as rotational symmetry and groups of rods we can make a bicycle wheel which work at maximum efficiency, we can also study equilibrium of forces practically and other applications.**

Keywords: **Pendulums, Bicycle wheel, oscillation, transitional and random motion.**

I. INTRODUCTION

When I ask you about beginning of bicycle's industry, your answer depend on from where you is? You will tell me that a person from your country invent it .but, at fact evolving bicycle take place by combination of different ideas from different nationalities. Up to 〖19〗^th century, construction of bicycles depend on comparison only – a tire attached to some spokes which attached to spindle... and so – after that a design which is insignificantly improved on with existing technology has introduced but until that time no one know how it behave, it's only an object which move although it's a very important machine, so structural engineers try to understand it's nature." Sharp(1977), in his original 1896 work, produce an analysis, based on the polygon of forces ,relating spoke tension to rim compression, but this was largely limited to the case of the unloaded wheel. A more thorough understanding of the properties of spooked wheels was made by Pippard and white (1932). This work was primary aimed at producing an understanding of light wheels for aircraft. Pippard couldn't analyze the behavior of the individual spokes; at the suggestion of south well he idealized the spokes as being disks with the property that stress could be carried only in one direction, he was then able to produce continuous function for the spoke tension, which can be compared with the discrete values measured in experiments. Two other studies are worthy of note. Bradt (1981) analyzed a wheel using a finite-element analysis, and Ivey (1985) loaded a complete bicycle through the seat in order to determine the stresses in a wheel. Both gave results that are broadly in agreement with the principles quoted previously, but neither result was compared with any other work. the finite-element analysis looked only at single point load applied to one point in the rim (thus ignoring any load-spreading from the tire) and assumed that the rim was made from straight elements between spoke positions, while then test was carried out on a bicycle with inflated tires and there is considerable scatter in the results" .

For pendulums, Italian scientist Galileo Galilei was the first to study the properties of pendulums, beginning around 1602; in 1656 the Dutch scientist Christiaan Huygens built the first pendulum clock. also, there are Huygens' Horologium Oscillatorium(1673), Temperature compensated pendulums(1721), the mercury pendulum in (1721) and the gridiron pendulum in (1726),Barton pendulum, Blackburn pendulum, Doubochinski pendulum, Double inverted pendulum, Double pendulum, inertia wheel pendulum, inverted pendulum, kapitiza pendulum, pendulum clock, pendulum mathematics, pendulum rocket fallacy, seconds pendulum, spherical pendulum…etc.

In my papers, bicycle isn't a mean transport, I deal with it when it is turned up down and it's wheel move freely, what will be the results if we apply classical mechanics laws? It will be a pendulum!

II. BASIC CONSTITUTION

The bicycle's pendulum constitution is similar to the general constitution of the bicycle's wheel, they consist basically of mass which has a circular shape, Spokes or rods attach mass with the spindle "rub" which lies at perpendicular level on mass and spokes, the whole system is based on a supporter. Figure 1 explains these components clearly.

III. CONSTRUCTION OF BICYCLE'S PENDULUM

The method of constructing bicycle's wheel is generally similar to constructing bicycle's pendulum, but simple differences take place as any rod in any quarter must have another equivalent rod in the opposite, equivalent rods means that these rods have same masses, lengths and angles with axes, if these rods isn't equivalent, the performance of pendulum won't be completely right, number of these rods is divisible on 4, it follow the mathematical series $(4,8,12,16,\ldots,m)$ where m is the max number of rods which can be built in the wheel, Also distribution of these spokes must be regular so that regular distribution of mass with respect to them takes place so that pendulum behave efficiently. If we construct the pendulum following the method of constructing bicycle in addition to the previous conditions we will obtain figure 2 which is a very simple model of bicycle's pendulum. We apply known conditions of equilibrium with the aid of figure 3 to assure that the new construction is right:

$$\sum_{1}^{4} F(X) = 0$$

$$\sum_{1}^{4} F(Y) = R - F_g + R - F_g + R - F_g + R - F_g = 0$$

$$\sum_{1}^{4} \tau = -F_g d + F_g d - F_g d + F_g d = 0$$

FIG.1.BASIC CONSTITUTION

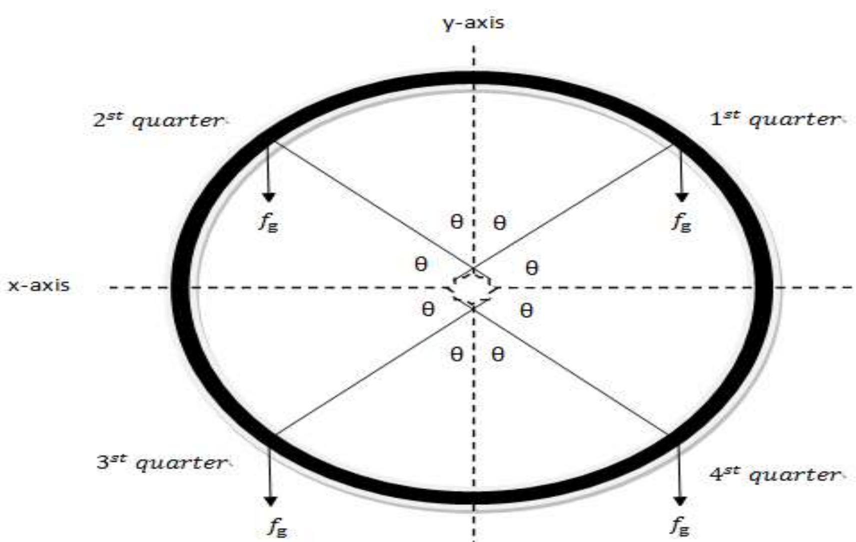

FIG.2.CONSTRUCTION OF PENDULUM

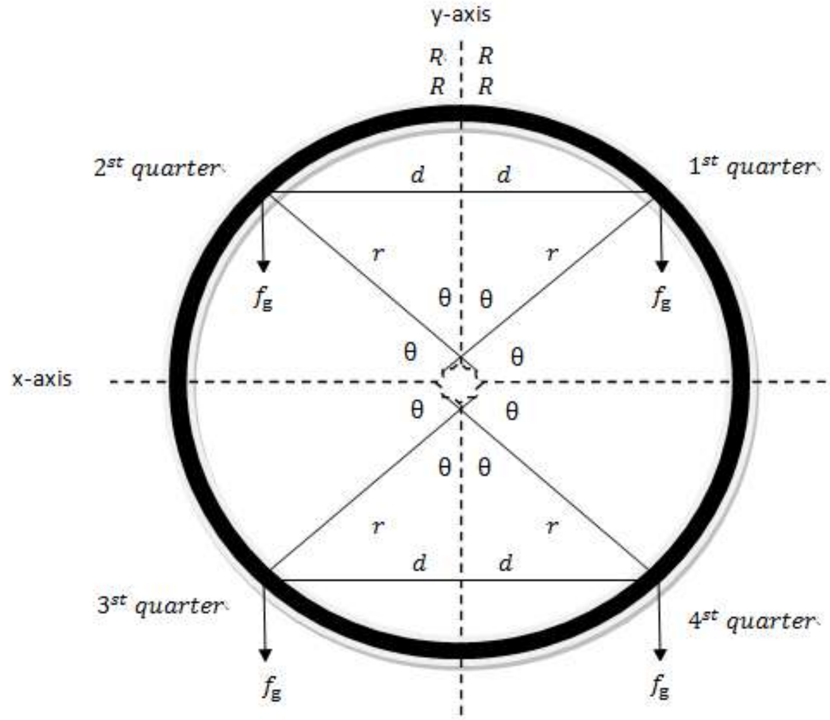

FIG.3.APPLING CONDITIONS OF EQUILIBRUIM

If we applied last conditions in general pendulums, equations will become:

$$\sum_{1}^{n} F(X) = 0$$

$$\sum_{1}^{4} F(Y) = R_1 - F_{g_1} + R_1 - F_{g_1} + R_1 - F_{g_1} + R_1 - F_{g_1} + R_2 - F_{g_2} + R_2 - F_{g_2} + R_2 - F_{g_2} + R_2 - F_{g_2} + \cdots - \cdots + R_n - F_{g_n} + R_n - F_{g_n} + R_n - F_{g_n} + R_n - F_{g_n} = 0$$

$$\sum_{1}^{n} \tau = -F_{g_1} d_1 + F_{g_1} d_1 - F_{g_1} d_1 + F_{g_1} d_1 - F_{g_2} d_2 + F_{g_2} d_2 - F_{g_2} d_2 + F_{g_2} d_2 - \cdots + \cdots - \cdots + \cdots - F_{g_n} d_n + F_{g_n} d_n - F_{g_n} d_n + F_{g_n} d_n = 0$$

Don't mix between the last numbers because it's refer to number of groups only -each 2 spokes and their opposite make a group- so, $R_1 = R_2 = R_2$ and the same for F_g and d.

IV. HOW THE PENDULUM WORKS

***Practical Experiments*:** Initially. The pendulum is at static case, we mark one rod of the pendulum and called it "referenced rod, suppose that this rode lies at –ve y-axis. displace the pendulum sideways so that the referenced rod make an angle θ with y-axis whatever this angle is right or left, wait and observe what happen.

***Theoretical Explanation*:** after displacing the pendulum side away, clockwise or counterclockwise. Non-equilibrium state takes place, so it try to return again to the state of equilibrium, during that, It makes back and force motion with meaning that the referenced rod move in both sides of the –ve y-axis but the displacement angle become less and less until the pendulum stop finally at the initial position. The pendulum returns to this position whatever the external force effect it. And coming words indicate these forces which can affect in the same direction of motion or in opposite direction**.**

***Practical Experiments*:** The pendulum is at static case. First: Affect the pendulum by a force of small magnitude "push it by your hand", Wait until it stop. Second, affect the pendulum by a force with large amplitude and wait again until it stops. Third: make the pendulum to move, affect a force during this motion and wait until it stops. Fourth: affect a force which makes the pendulum stop when the referenced rod is at equilibrium location. At each time observe what happen before stopping.

***Theoretical Explanation*:** at each time we find that the pendulum return finally to the equilibrium state by making back and forth motion and the amplitude become less and less until it stop and if we did the latter experiment the pendulum doesn't move because it's already at equilibrium state.

V. PROPERTIES OF BICYCLE'S PENDULUM

Distribution of rods: I mentioned before that rods have regular distribution but, there are several types of distribution, it can be one by one, twofold, threefold, fourfold…etc. so dealing with groups of spokes is better, distribution of these groups, is regular so, angles among them are equals, also, angles among spokes in each group are equals.

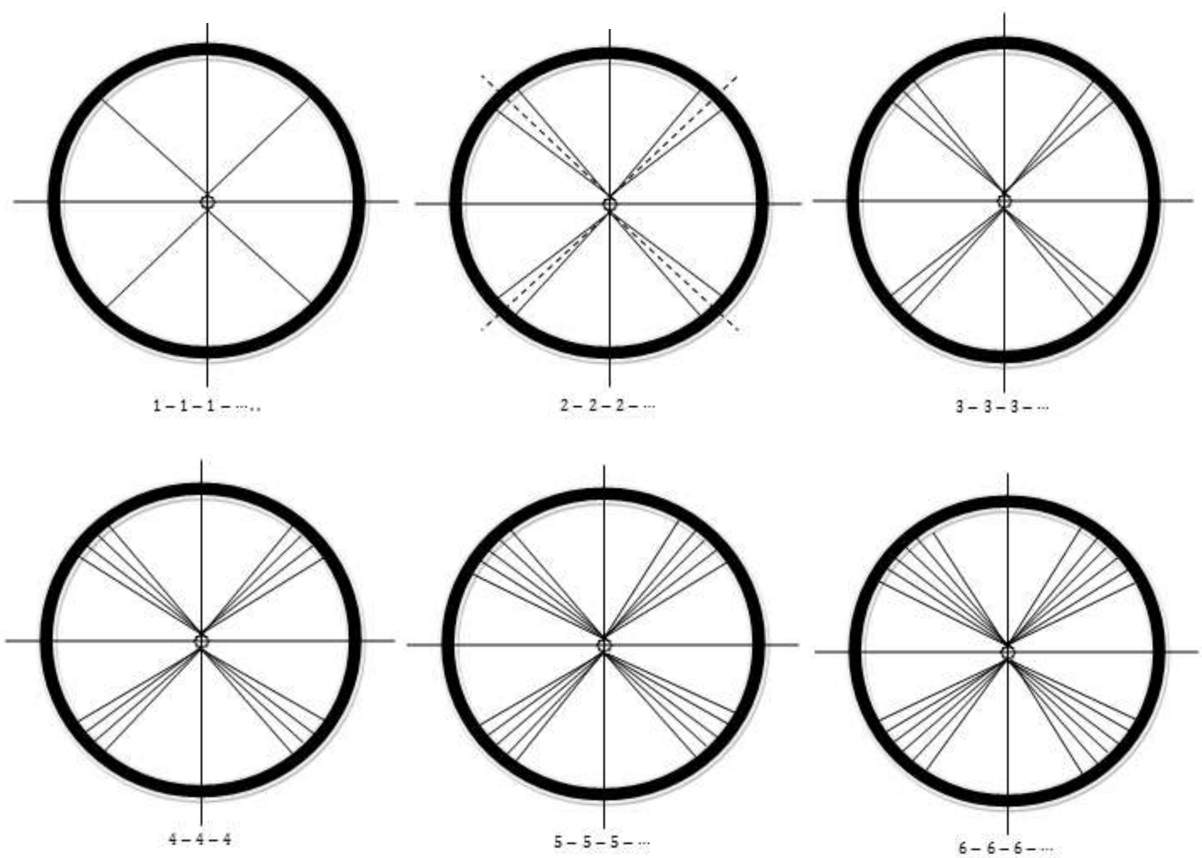

FIG.5.DISTRIBUTION OF GROUPS IN

Rotational Symmetry: bicycle's pendulum can make a big number of rotational symmetry because it has a lot of spokes, number of rotational symmetry equals to number of its spokes "rods" as shown in the coming figures where we find that number of rods is 4 and order of symmetry is 4 also but to have better expression we say that rotational symmetry order equals to number of groups.

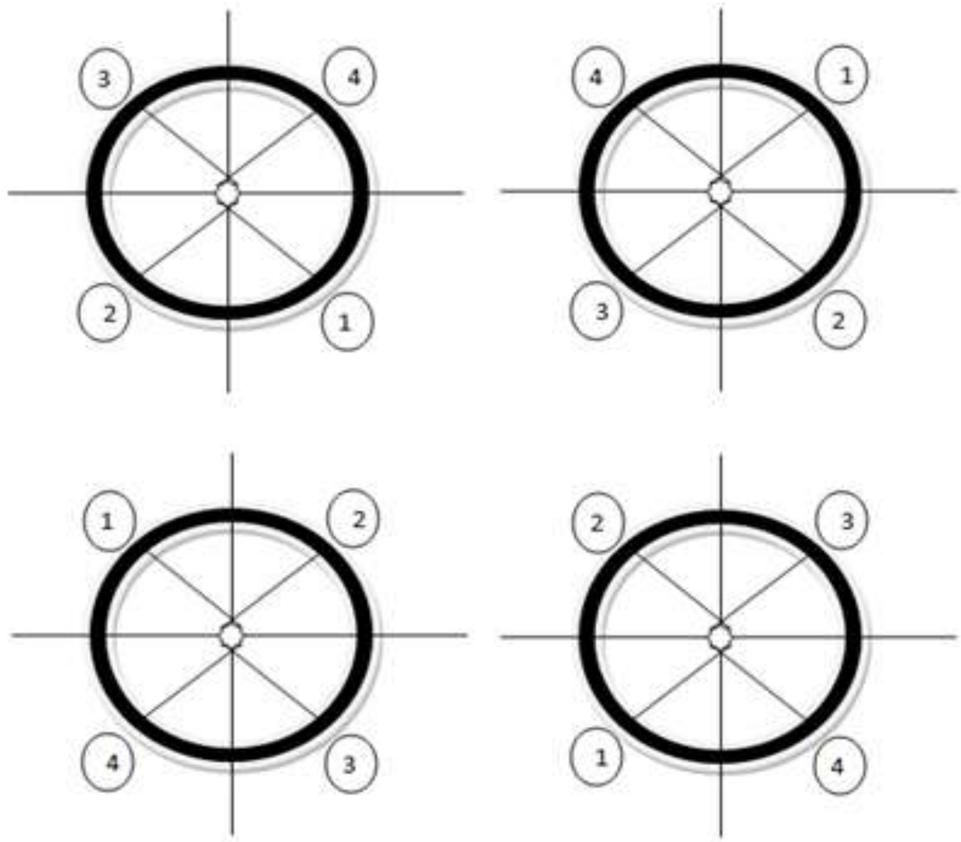

FIG. 6 ROTATIONAL SYMMETRY

The least number of rods in bicycle's pendulum equals to 4 but theoretically we can say that it equals to 2rodsbut the pendulum works at bad efficiency in this state so, it's better that there is at least one rod in each quarter, hence number of rods equals to 4 rods.

Number of referenced points: We mentioned before that the referenced point refer to the rod which we mark and deal with. And the location of them is reference locations but how many points and locations I can deal with?, In fact, number of these points equal to number of rods because I can mark all them but number of referenced locations is the double of rods' numbers because if we rotate the wheel with 180°, we find that the pendulum stop, so other points become in same locations.
Also types of motion and time of full oscillation are properties of the pendulum.

VI. TYPES OF MOTION

***Practical Experiments*:** when the pendulum at static case and we affect a force, it rotates. Observe the motion of the pendulum until it stops.

***Theoretical explanation**:* after we affect a force, the pendulum rotates about its Spindle. But, its angular velocity reduce with time until the pendulum can't make complete turn .at that time, it start to oscillate. But, because of angular velocity reduction with time, the amplitude reduces. But, this reduction isn't big so that the pendulum let a small displacement on the path of its motion without moving in. Hence there are four types of motion in the pendulum: rotational motion, oscillation motion, translation motion and random motion, coming words and figure 7 explain that obviously.

***First: rotational motion*:** it is the first observed motion in the pendulum; it takes place after effect a force of large magnitude so that the bicycle can make completed turns. But, as I mentioned before that the angular velocity reduce with time. So, bicycles' pendulum makes non-uniform rotational motion.

***Second: oscillatory motion*:** as we know that the oscillation motion is the motion which repeat itself frequently with meaning that it move with some points more than once, and I mentioned before that the angular velocity reduce until it make less than one turn. But, it doesn't stop, it make back and forth motion. So, it moves with some points more than one. Hence it makes oscillatory motion. But the range of it reduces with time until it stops finally. Oscillation motion is represented in figure 7 with red.

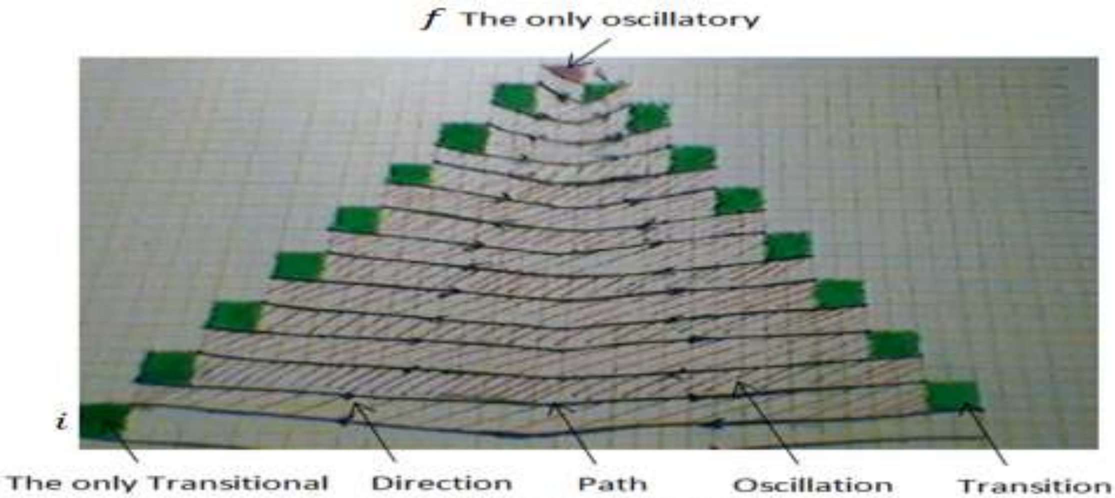

FIG.7.TYPES OF MOTION

Third transitional motion: as it mentioned in oscillatory motion that the range of it reduce with time because of the repeated points range reduction but if we focus in the new range of points which doesn't represent oscillatory motion in its new oscillation, we find that it sometimes can be represented by a straight line and we know that the translation motion is the movement of an object from one point to another point at straight line. Hence, the pendulum make transitional motion on rotational path, it represented in figure 7 by green.

Fourth random motion: we know that random motion take place when the body makes uncontrolled motion so, any object in the world can make random motion because it can move without controlling, and peoples themselves sometimes make random motion. Hence, bicycle's pendulum makes random motion.

Comments on types of motion: if we focus on pendulum's path, we find that there is only one area of oscillatory motion which won't be transitional "f", also there is only one area of translation which wasn't oscillatory"i".

VII. THE TIME OF FULL OSCILLATION

***Practical Experiments* :** when the pendulum is at static case**,** we displaced the pendulum side away so that the marked rods make an angle $\theta$with the initial location of it**,** prepare the stopwatch and start to measure time when it start to return **,** wait until the referenced rod pass with the initial position and continue its motion to the opposite direction until it stop and return back passing with the initial location again, continuing motion and when it stop, stop the stopwatch and record the reading of stopwatch.

Theoretical explanation: the time of full oscillation is similar to the periodic time. We know that the time of full oscillation is the time required to make a completed turn or the time taken by the object to pass with a point twice in the same direction.

But in bicycle's pendulum we find that the time of full oscillation is the time taken by the marked rod between 2 successive pausing at the same side as shown in figure.

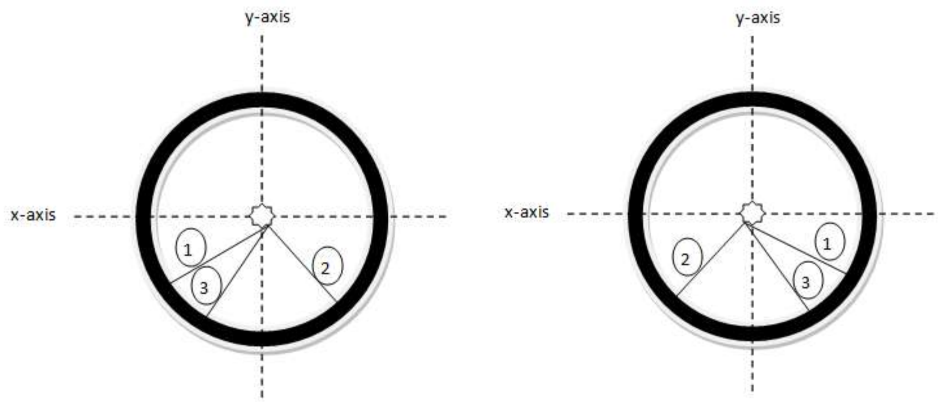

FIG. 8 TIME

Law: After a lot of experiments I can obtain the law of full of oscillation

$$T = n\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\left(\sin\frac{\theta}{2}\right)^2 + \frac{9}{64}\left(\sin\frac{\theta}{2}\right)^4 + \cdots\right)$$

$n \propto u$

Where n is constant, u is coefficient of friction in spindle.

Relation between the time of full oscillation and some variables

In all our experiments we will deal with a pendulum which has coefficient of friction sufficient to make constant $n \approx 15.6$

First: the relation between the time of full oscillation "T" and the length of the rod"L"

***Practical Experiments*:** we use a pendulum of length "l"**,** calculate the time of full oscillation, replace this pendulum by another with length L , calculate the time of full oscillation again and compare between the 2 values of them

Theoretical explanation: from the law of time of full oscillation

$$T = n\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\left(\sin\frac{\theta}{2}\right)^2 + \frac{9}{64}\left(\sin\frac{\theta}{2}\right)^4 + \cdots\right)$$

We find that the time of full oscillation is directly proportional to the root of rod's length

$$T \propto \sqrt{l}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}$$

Second: relation between the time of full oscillation "T" and the angle "θ"

***Practical Experiments*:** when the pendulum at static case, make the marked rod have an angle α with the y-axisandcalculate T**.** Repeat these steps for several angles$\beta, \gamma, \delta, \theta, \varphi, \dots etc$ make relation between the time of full oscillation and angle of rotation.

Theoretical explanation: From the law

$$T = n\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\left(\sin\frac{\theta}{2}\right)^2 + \frac{9}{64}\left(\sin\frac{\theta}{2}\right)^4 + \cdots\right)$$

Where θ is the maximum angle.

We find that the time of full oscillation increase by the increase of the angle until it reaches to 180◦. After that, the time of full oscillation decrease by the increase of the angle until it reaches to 360◦, and so as shown in figure 9.And table 1 where we used a pendulum with "$L = .32m$" and particular value of coefficient of friction which makes"n" equals to 15.6

Table.1.change time of full oscillation with change of angles

θ	T"theoritically	T"particularly1	T"particularly
27	7.3	7.11	6.18
54	7.46	7.30	7.21
81	7.69	7.65	7.62
108	8.33	8.29	8.35
135	9.09	9.21	8.91
162	9.44	9.52	9.45
189	9.73	9.81	9.86
216	9.98	10.05	9.94
243	9.73	9.82	9.71
270	9.44	9.47	9.56
297	9.09	9.21	8.95
324	8.33	8.45	8.47
351	7.69	7.78	7.72

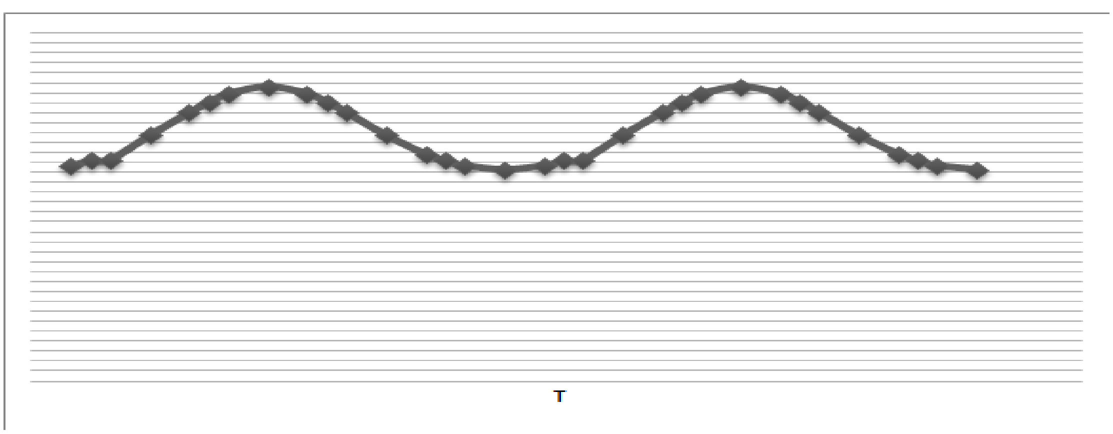

FIG.9 CHANGE IN TIME OF FULL OSCILLATION WITH ANGLES VALUES

But when θ is small, we find that the time of full oscillation is the same approximately at different angles as shown in table 2 and we could use the law of time of full oscillation as

$$T = n\,\pi\sqrt{\frac{l}{g}}$$

And for a bicycles which has features as the last bicycle which use to fill last table we find that

$$T = 15.6\,\pi\sqrt{\frac{l}{g}}$$

Table.2.change of time of oscillation values with angles of small values

θ	T
30	8.675
29	8.665
28	8.656
27	8.646
26	8.638
25	8.629
24	8.621
23	8.613
22	8.606
21	8.599
20	8.592
19	8.586
18	8.580
17	8.574
16	8.568
15	8.563
14	8.559
13	8.554
12	8.550
11	8.546
10	8.543
9	8.540
8	8.537
7	8.535
6	8.533
5	8.531
4	8.529
3	8.528
2	8.5278
1	8.5273

Third: relation between time of full oscillation and number of rods

Practical Experiments: use a pendulum with number of rods$N_1 = 32$, calculate$T$, replace the pendulum by another with number of rods $N_2 = 36$ for examples and calculate T again. Finally, compare between the 2 digits.

Theoretical explanation: from the law of the time of full oscillation we find that the time of full oscillation doesn't depend on the number of rods in the pendulum.

The condition of measuring the bicycle's pendulum:

To measure the time of full oscillation, the pendulum must rotate without any external force with meaning that if we affect by a force, we must wait until the effect of this force finished. And after that we start to determine the time of full oscillation by the last laws.

BICYCLE AND ROTATIONAL MOTION

Aw we know that bicycle make rotational motion when we drive it on roads but if we make it's wheel to move freely we find that it make several types of motion as rotational motion, oscillation motion, transitional motion and random motion and the explanation of that is mentioned in part of "types of motion"

VIII. APPLICATIONS OF BICYCLE'S PENDULUM

1- tires "wheel" industry

We can use this pendulum to increase the efficiency of the tires and make sure that it works at maximum efficiency

Method: we displace the tire sideways and wait until it stops, if the tire return to its initial location hence it work with maximum efficiency but if it isn't return to the initial location hence we must repair it. Also, when full oscillation time values are similar to last measured calculus of your pendulum, hence it works at with maximum efficiency.

2-study in the forces at equilibrium

We can use this pendulum to study forces at equilibrium using the idea of returning to the initial location "equilibrium location"

Method:-when the pendulum is static and we put a mass at any position "it's rod make an angle θ with the referenced rod", the pendulum will move and the referenced location change, if we put another mass with the same magnitude at opposition side "the rod makes the same angle with the referenced rod ", we find that the pendulum returns to the referenced location "the location which take place before adding any mass".

3-deremining acceleration of gravity

We could use it to determine the acceleration of gravity

The idea: From the law of period

$$T = n\,\pi \sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\left(\sin\left(\frac{\theta}{2}\right)\right)^2 + \frac{9}{64}\left(\sin\left(\frac{\theta}{2}\right)\right)^4 + \cdots\right)$$

We can easily calculate the length, period, angle of rotation, and after compensation in the law, we can calculate the value of gravity of acceleration.

Although there are a lot of methods used in determining the gravity acceleration as simple pendulum. But, my pendulum can work in 360°, it is reversible and it can play in various levels.

IX. CONCLUSION

Bicycle's wheel is a pendulum which consists basically of mass, rods and spindle, we can construct it easily using the method of building bicycle but we can build simples shape because number of rods is divisible on 4 so the least number of them is 4, when you displace this pendulum at any direction, you finds that it returns to its initial location again and you can assure from that by marked one rod and call it referenced rods, also you can deal with all rods of the pendulum as referenced rods, rods of the pendulum is distributed regularly but this regular distribution become better when you deals with groups of rods and one group have 1 rod or 2 or 3…etc., angles among groups are equals, also angles among rods of group are equals and number of these groups equals to rotational symmetry order. When we look at the nature of the pendulums motion we find that it make rotational motion, oscillatory motion, transitional motion and also random motion, the time request to make full oscillation by the pendulum can be measured by a law which give us relation between it and length of rods, angle o oscillation and number of rods, using this time and pendulum properties we can make bicycles which work at maximum efficiency, study equilibrium of forces easily and other applications.

APPENDIX

$R = reactionduetoweightoftire'sparts$
$f_g = tire'spartweight$
$d = perpendiculardistancebetweenweightandreaction$
$\theta = anglebetweenspokeandit'sinitiallocation$
$r = radius, nearlylengthofspoke$
$M_O = momentaboutspindle$
$H: angularmomentumaboutspindle$
$I = momentofinertia$
$\alpha = angularacceleration$
$T = timeoffulloscillation$

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